Reports of the Department of Geodetic Science

Report No. 93

# DATA ANALYSIS IN CONNECTION WITH THE NATIONAL GEODETIC SATELLITE PROGRAM

by

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# PREFACE

This project is under the technical direction of Jerome D. Rosenberg, Project Manager of the National Geodetic Satellite Program at NASA Headquarters, Washington, D. C. The contract is administered by the Office of Grants and Research Contracts, Office of Space Science and Applications, NASA, Washington, D. C.

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## 1. STATEMENT OF WORK

Conduct the first two years of a multi-year study and analysis of data from satellites launched specifically for geodetic purposes and from other satellites useful in geodetic studies. The program of work follows the proposal #36-008(033) and includes analysis of positions derived from photographic observations of both reflecting and emitting satellites, from range observations, and from any other suitable type of observations.

The final analysis will be an improved map placing all participating tracking stations on a single worldwide coordinate system. In deriving the final results, The Ohio State University representatives work with other representatives from universities and government agencies to prepare a single handbook compiling the best geodetic data available at the time.

The work includes, but is not limited to, the following:

(1) Programming, testing, debugging of analysis methods,

(2) Participating in working groups and other planning meetings to establish desirable operational procedures including tracking procedures, participants' selection, data format, communication procedures, analysis procedures, etc.,

(3) Providing advice to NASA on various aspects of the National Geodetic Satellite Program,

(4) Making use of the available observational data to determine relative positions of observation stations in an arbitrary Cartesian coordinate system and then obtaining geocentric coordinates for these stations.

## 2. TECHNICAL REPORTS PUBLISHED

Detailed information on the results of the investigation in connection with the project are to be found in the following Reports of the Department of Geodetic Science, The Ohio State University:

70 "The Determination and Distribution of Precise Time" by H.D. Preuss, April 1966

71 "Proposed Optical Network for the National Geodetic Satellite Program" by I.I. Mueller, May 1966

82 "Preprocessing Optical Satellite Observations" by F.D. Hotter, April 1967

86 "Least Squares Adjustment of Satellite Observations for Simultaneous Directions or Ranges. Part I: Theory" by E.J. Krakiwsky and A.J. Pope, in press

87 "Least Squares Adjustment of Satellite Observations for Simultaneous Directions or Ranges. Part II: Computer Programs" by E.J. Krakiwsky, G. Blaha and J. Ferrier, in press

88 "Least Squares Adjustment of Satellite Observations for Simultaneous Directions or Ranges. Part III: Computer Program Subroutines" by E.J. Krakiwsky and J. Ferrier, in press

A summary of the findings is presented below in section 3.

## 3. INVESTIGATIONS IN CONNECTION WITH THE GEOMETRIC ANALYSIS OF GEODETIC SATELLITE DATA

### 3.1 Primary Objective

The primary objective of the OSU investigation is the geometric analysis of geodetic satellite data. The analysis is accomplished in three steps:

(1) The establishment of a primary network where station positions are known to an internal consistency of approximately 10 meters or better to serve the following purposes: (a) unify the various geodetic datums in use around the world, (b) connect NASA tracking stations, isolated islands, navigational beacons, and other points of interest to the unified system.

(2) Establishment of a densification network where station positions are known to an internal consistency of approximately 3 meters or better to serve the following purposes: (a) improve the internal quality of existing geodetic systems (triangulation, etc.) by establishing "super" control points in sufficient number, (b) to provide control for mapping to scales as large as 1 : 24,000.

(3) Establishment of a set of scientific reference stations where positions are known to an accuracy of one meter or better with respect to the unified system for advanced applications.

### 3.2 Accomplishments During the Report Period

#### 3.21 Planned Geodetic Networks

The original network as proposed to NASA and presented at the 47th annual meeting of the American Geophysical Union in 1966 aimed at (i) the connection of the major geodetic datum blocks shown in Fig. 1, (ii) the derivation of a common geocentric-geodetic datum, and (iii) tying the NASA-supported tracking stations (Fig. 2) to this world datum. This network is shown in Fig. 3. The plan includes the ESSA-DOD primary geometric world triangulation net with its co-located TRANET and SECOR stations. The underlying philosophy of the proposed network was to tie the supplementary sites to this relative primary geometric world net, and then connect this to a number of "absolute" stations where satellites were observed through an extended period of time. Through this procedure the coordinates of all stations involved could be determined in a geocentric earth-fixed coordinate system. Scaling was to be achieved by available SECOR measurements and by precise terrestrial baselines in Australia, Europe, and in the USA.

During the interim period since April, 1966, certain additional requirements arose, such as the provisional updating of the Mercury datum (derived in 1959) on which most NASA tracking stations are located and the positioning of remote stations with no ties to this datum and of the Loran-C navigational beacons. These requirements necessitate minor changes and additions to the original plan.

### 3.22 Treatment of the Observation Data

#### 3.221 Optical Data

The procedure to obtain the appropriate coordinates of the satellite from its photograph taken with a background of stars, followed to some extent by most observer-groups participating in the program, is the following:

(i) The stars' coordinates, from their "mean" catalogued positions to their "observed" positions, are updated as shown in Fig. 4.

In the figure the symbols $\underline{R}_i(\theta)$ denote rotation matrices of 3 x 3 dimension. The elements $r_{lm}$ of the matrices satisfy the following rules: $r_{ii} = 1$; $r_{ij} = r_{ji} = r_{ki} = 0$; $r_{jj} = r_{kk} = +\cos\theta$; $r_{jk} = +\sin\theta$; $r_{kj} = -\sin\theta$; where $j \equiv i$ (modulo 3) + 1, $k \equiv j$ (modulo 3) + 1. These rules are consistent with a right-handed coordinate system and positive signs for counterclockwise rotation, as viewed looking toward the origin from the positive axis.

## MAJOR GEODETIC DATUM BLOCKS

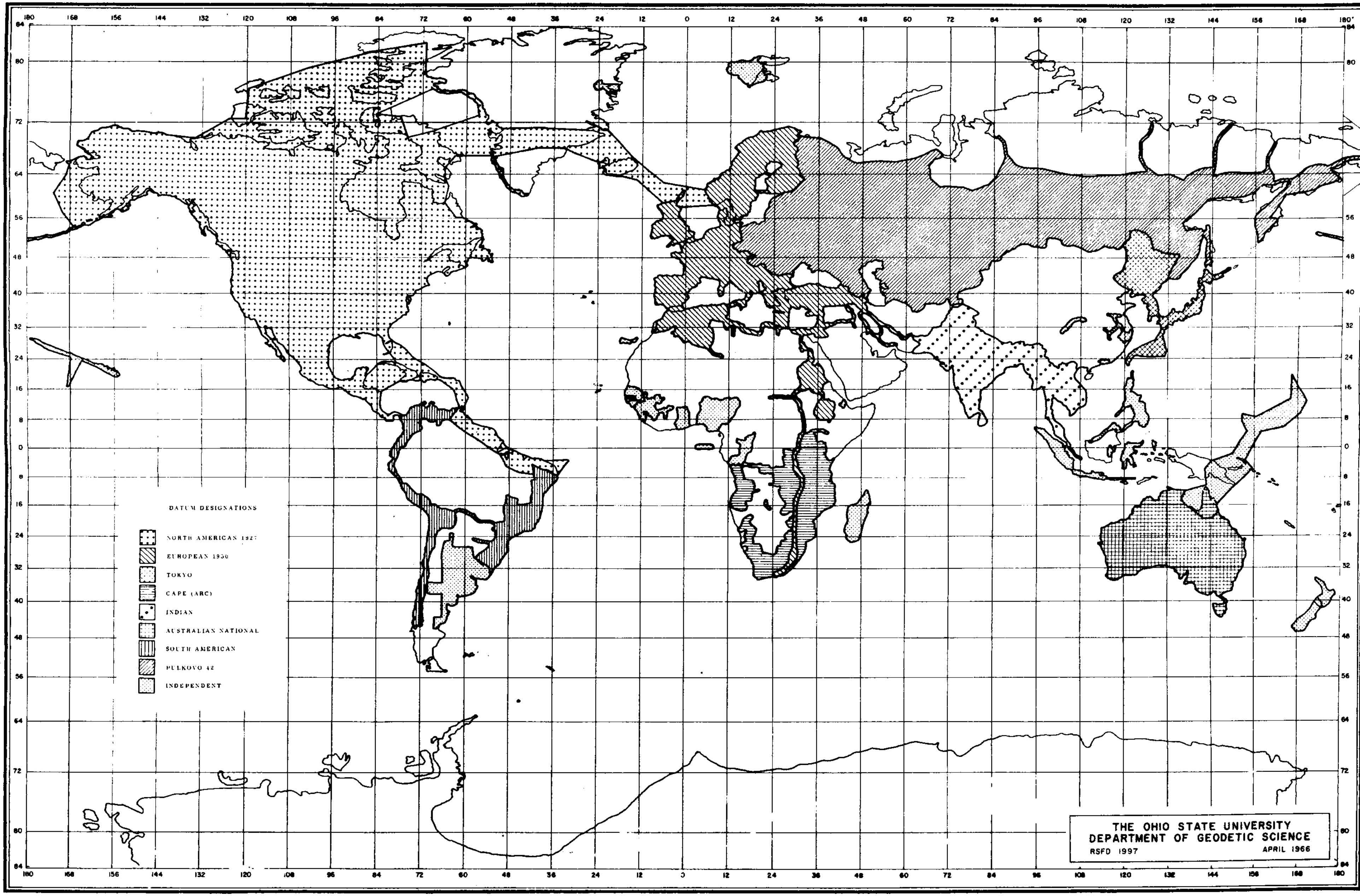



FIGURE I

# NASA SUPPORTED STATIONS

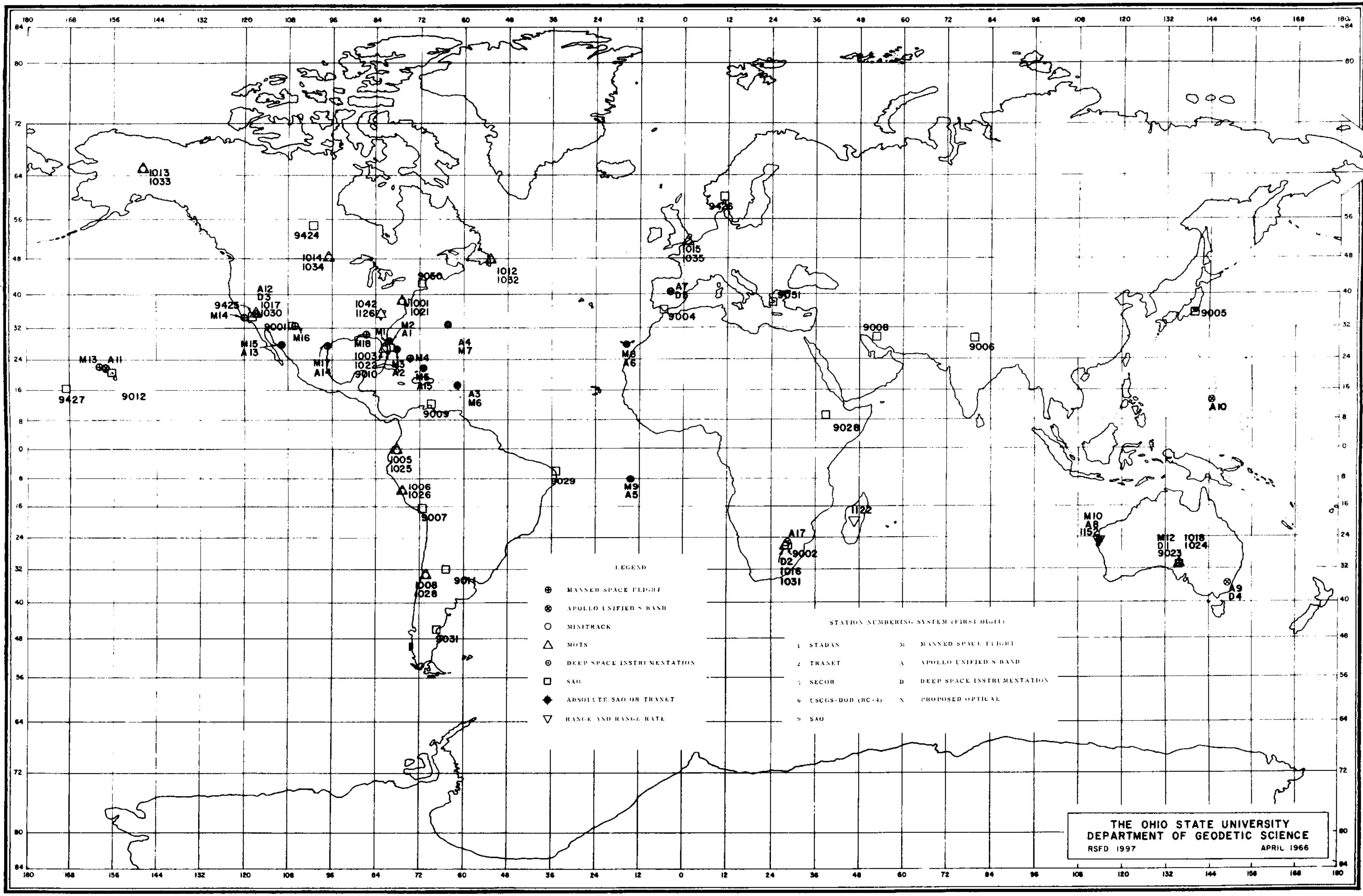
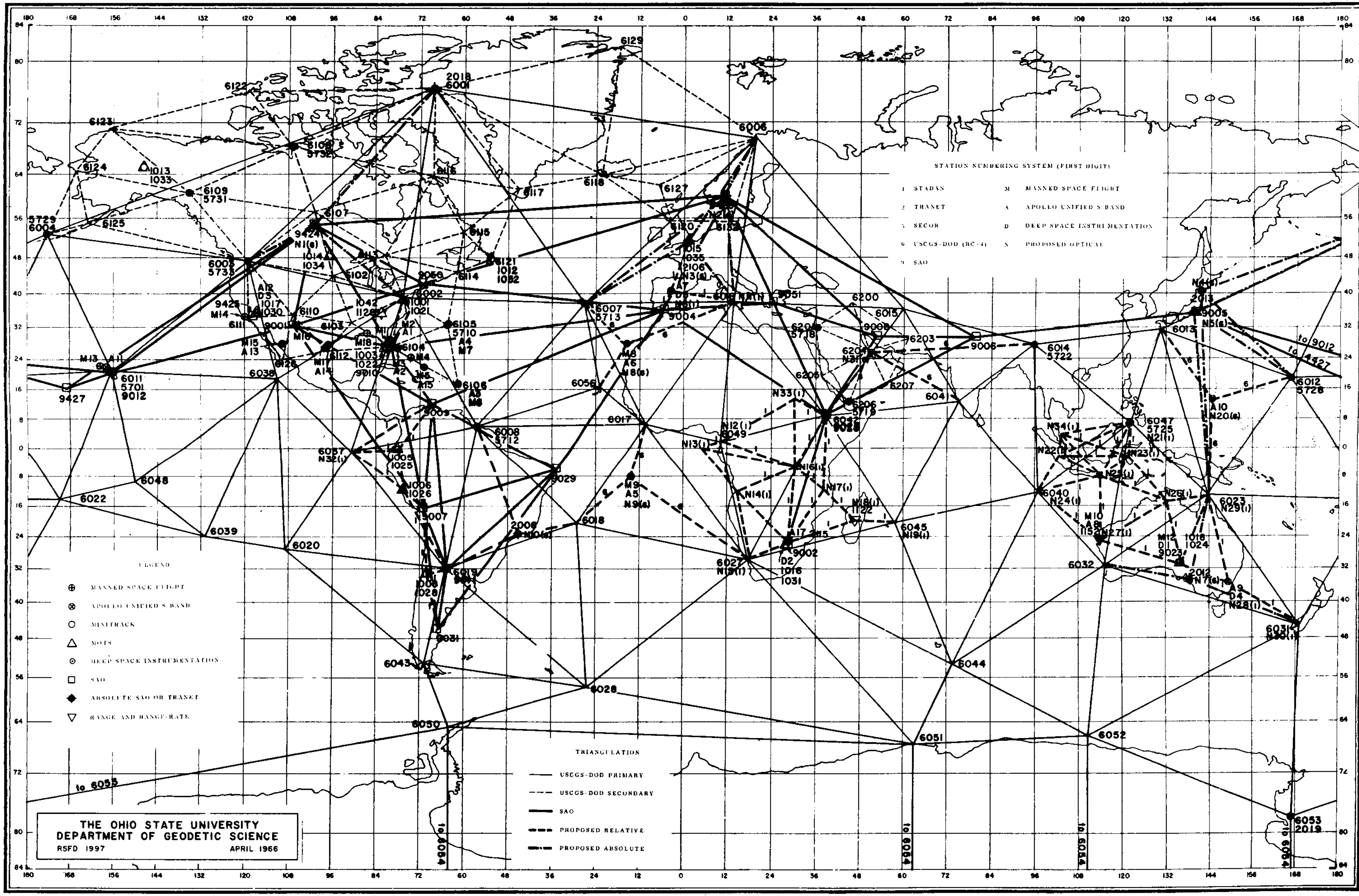



FIGURE 2

PROPOSED OPTICAL NETWORK




Stations selected for planning purposes only.

FIGURE 3

# STAR UPDATING

**CATALOGUED (HELIOCENTRIC)** $\alpha_0, \delta_0, (T_0)$

**CATALOGUED** $X_0, Y_0, Z_0$ →

$$\begin{pmatrix} X_0 \\ Y_0 \\ Z_0 \end{pmatrix} = \begin{pmatrix} \cos\delta_0 \cos\alpha_0 \\ \cos\delta_0 \sin\alpha_0 \\ \sin\delta_0 \end{pmatrix}$$

**PROPER MOTION** →

$$\underline{M} = \underline{R}_3(-\alpha_0)\underline{R}_2(\delta_0-90°)\underline{R}_3(\psi_0)\underline{R}_2(\mu t + \tfrac{1}{2}\tfrac{d\mu}{dt}t^2)\underline{R}_3(-\psi_0)\underline{R}_2(90°-\delta_0) + \underline{R}_3(\alpha_0)$$

**PRECESSION** →

$$\underline{P} = \underline{R}_3(-z)\underline{R}_2(\theta)\underline{R}_3(-\zeta_0)$$

**NUTATION** →

$$\underline{N} = \underline{R}_1(-\epsilon-\Delta\epsilon)\underline{R}_3(-\Delta\psi)\underline{R}_1(\epsilon)$$

**ANNUAL ABERRATION** →

$$\underline{A} = \begin{pmatrix} -D \\ C \\ C\tan\epsilon \end{pmatrix} = \underline{R}_3(-f')\underline{R}_2(A')\underline{R}_1(B') \begin{pmatrix} -D' \\ C' \\ C'\tan\epsilon \end{pmatrix}$$

**ANNUAL PARALLAX** →

$$\underline{R} = -\begin{pmatrix} C\sec\epsilon \\ D\cos\epsilon \\ D\sin\epsilon \end{pmatrix} \frac{\pi}{\kappa} = -\underline{R}_3(-f')\underline{R}_2(A')\underline{R}_1(B') \begin{pmatrix} C'\tan\epsilon \\ D'\cos\epsilon \\ D'\sin\epsilon \end{pmatrix}\frac{\pi}{\kappa}$$

**APPARENT** $X, Y, Z$ →

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \underline{R} + \underline{A} + \underline{N}\,\underline{P}\,\underline{M} \begin{pmatrix} X_0 \\ Y_0 \\ Z_0 \end{pmatrix}$$

**APPARENT (GEOCENTRIC)** $\alpha, \delta, (T)$ →

$$\alpha = \tan^{-1}\frac{Y}{X}$$
$$\delta = \sin^{-1}Z = \tan^{-1}\frac{Z}{\sqrt{X^2+Y^2}}$$

**DIURNAL ABERRATION** → $\Delta\alpha_A, \Delta\delta_A$ (CONVENTIONAL)

**STELLAR REFRACTION** → $\Delta\alpha_R, \Delta\delta_R$ (CONVENTIONAL)

**OBSERVED (TOPOCENTRIC)** $\alpha^*, \delta^*, (T)$ →

$$\alpha^* = \alpha + \Delta\alpha_A + \Delta\alpha_R$$
$$\delta^* = \delta + \Delta\delta_A + \Delta\delta_R$$

NOTATION:

| | |
|---|---|
| $\psi_0, \mu$ | DIRECTION AND MAGNITUDE OF PROPER MOTION AT $T_0$ IN THE MEAN COORD. SYSTEM |
| $t = T - T_0$ | |
| $z, \theta, \zeta_0$ | PRECESSIONAL ELEMENTS FOR THE INTERVAL $t = T - T_0$ |
| $\Delta\epsilon, \Delta\psi$ | NUTATION IN OBLIQUITY AND LONGITUDE AT T. |
| $\epsilon$ | MEAN OBLIQUITY OF THE ECLIPTIC AT T. |
| C, D | BESSELIAN DAY NUMBERS AT T IN THE TRUE COORDINATE SYSTEM |
| A', B', C', D' | BESSELIAN DAY NUMBERS AT THE BEG. OF THE B.Y. NEAREST TO T IN THE MEAN COORDINATE SYSTEM (AS TABULATED IN THE EPHEMERIS) |
| f' | INDEPENDENT DAY NUMBER AT THE BEGINNING OF THE B.Y. NEAREST TO T IN THE MEAN COORDINATE SYSTEM |
| $\pi$ | ANNUAL PARALLAX |
| $\kappa$ | CONSTANT OF ABERRATION |

FIGURE 4

The symbols $\underline{P}_i$ denote permutation matrices of $3 \times 3$ dimensions. The elements $p_{lm}$ of the matrices are equal to zero except for $p_{ii} = -1$ and $p_{jj} = = p_{kk} = 1$.

The most advantageous catalogue to use at present is that of the SAO (Smithsonian Astrophysical Observatory), which is in the FK4 fundamental system, contains about 259,000 stars having an average distribution of six stars per square degree and an average standard deviation of about $\pm 0\overset{''}{.}5$ (at present).

(ii) From the updated stars' positions and their measured plate coordinates, the calibration parameters of the camera system and/or the plate constants are determined utilizing either photogrammetric (Fig. 5) or astrometric (Fig. 6, techniques.

(iii) Using these parameters and the measured plate coordinates of the satellite images the "observed" position of the satellite is calculated.

(iv) Appropriate corrections are finally applied to the "observed" satellite position to reduce it to the average terrestrial system (axes toward the IPMS 1900-05 average terrestrial pole, and the meridian of the BIH "mean observatory") as shown in Fig. 7, or to any other system in which the adjustment is performed when computing the station coordinates.

Actual procedures followed by the various participating groups may vary with respect to each other in terms of the constants, type, and number or corrections (e.g., the data should be "homogenized" or preprocessed. The procedures of the major U.S. agencies participating in the National Geodetic Satellite Program are shown in Fig. 8. The data as deposited in the GSDS in Greenbelt, Maryland, has been treated as shown. If, for example, the desired satellite position is the "true" (see Fig. 7), data preprocessing in the areas shaded in Fig. 8 is necessary. An example of what this could mean in terms of computational work is shown in Fig. 9.

3.222 Non-Optical Data

The other tracking systems utilized in the network are the Pulse-laser, the NASA Range / Range Rate, and the SECOR-range. At present, from the point

PHOTOGRAMMETRIC CALIBRATION

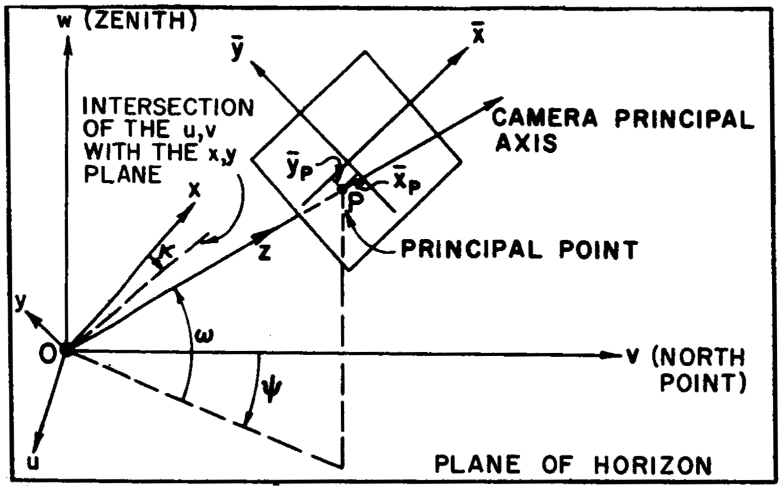
$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \underline{P}_1 \underline{P}_2 \underline{R}_1 (\Phi - \tfrac{\pi}{2}) \underline{P}_1 \underline{R}_3 (-\theta_L) \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_{\alpha, \delta \text{ (obs.)}}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \bar{x} - \bar{x}_P \\ \bar{y} - \bar{y}_P \\ c \end{pmatrix} = \bar{\kappa}\, \underline{P}_1 \underline{R}_3 (-\kappa) \underline{P}_2 \underline{R}_1 (\omega - \tfrac{\pi}{2}) \underline{R}_3 (-\psi) \begin{pmatrix} u \\ v \\ w \end{pmatrix}$$

$\Phi, \theta_L$ STATION LATITUDE, LOCAL SIDEREAL TIME

$\bar{x}_P, \bar{y}_P, c$ ELEMENTS OF INTERIOR ORIENTATION

$\kappa, \omega, \psi$ ELEMENTS OF EXTERIOR ORIENTATION




$$\left.\begin{aligned} \bar{x} &= \bar{x}' + \sum_{i=1}^{n} dx_i \\ \bar{y} &= \bar{y}' + \sum_{i=1}^{n} dy_i \end{aligned}\right\}$$ = MEASURED PLATE COORDINATES ( $\bar{x}', \bar{y}'$ ) CORRECTED FOR COMPARATOR ERROR, LENS DISTORTION, ETC.

COMPARATOR ERRORS (NOT CANCELLING):

i = 1 NONPERPENDICULARITY OF COMPARATOR AXIS

2 WEAVE OF GUIDE OF THE COMPARATOR AXIS

3 PERIODIC SCREW ERROR

4 SECULAR SCREW ERROR

LENS DISTORTIONS:

5 SYMMETRICAL RADIAL LENS DISTORTION

$$\left.\begin{matrix} dx_5 \\ dy_5 \end{matrix}\right\} = \left.\begin{matrix} \bar{x} \\ \bar{y} \end{matrix}\right\} (K_1 r^2 + K_2 r^4 + K_3 r^6 + \ldots\ldots)$$

6 TANGENTIAL LENS DISTORTION

$$\left.\begin{matrix} dx_6 \\ dy_6 \end{matrix}\right\} = \left.\begin{matrix} + \\ - \end{matrix}\right\} (J_1 r^2 + J_2 r^4 + \ldots\ldots) \begin{Bmatrix} \sin \hat{\phi} \\ \cos \hat{\phi} \end{Bmatrix}$$

⋮

n

<u>UNKNOWNS:</u>

$\bar{x}_P, \bar{y}_P, c$
$\kappa, \omega, \psi$
$K_1, K_2, K_3, J_1, J_2, \hat{\phi}$, etc. (MAY BE PREDETERMINED)
OTHERS: COEFFICIENTS IN THE REFRACTION FORMULA
STAR COORDINATES CONSTRAINED BY ST. ERROR GIVEN IN CATALOG

FIGURE 5

## ASTROMETRIC CALIBRATION

$$\left.\begin{aligned} \xi &= \frac{A\bar{x}' + B\bar{y}' + E}{G\bar{x}' + H\bar{y}' + I} \\ \eta &= \frac{C\bar{x}' + D\bar{y}' + F}{G\bar{x}' + H\bar{y}' + I} \end{aligned}\right\} \text{GENERAL TRANSFORMATION BETWEEN THE PLATE COORDINATES AND THE STANDARD COORDINATES}$$

WHERE

$$\left.\begin{matrix} \xi \\ \eta \end{matrix}\right\} = \left.\begin{matrix} \xi \\ \eta \end{matrix}\right\} (\alpha, \delta, \alpha_o, \delta_o) \rightarrow \text{STANDARD COORDINATES FROM } \left.\begin{matrix} \text{GNOMONIC} \\ \text{CYLINDRIC} \\ \text{OTHER} \end{matrix}\right\} \text{PROJECTIONS}$$

$\bar{x}', \bar{y}'$ MEASURED PLATE COORDINATES

VARIATIONS

1. $I = 1$
2. $I = 1$ AND $G = H = 0$ (LINEAR METHOD), THEN

$$\left.\begin{aligned} \xi &= A\bar{x}' + B\bar{y}' + E \\ \eta &= C\bar{x}' + D\bar{y}' + F \end{aligned}\right\} \text{AFFINE LINEAR TRANSFORMATION}$$

WHERE

$$\left.\begin{matrix} \left.\begin{matrix} A \\ C \end{matrix}\right\} = k_\xi \begin{Bmatrix} \cos\gamma \\ \sin\gamma \end{Bmatrix} \\ \left.\begin{matrix} B \\ D \end{matrix}\right\} = k_\eta \begin{Bmatrix} \sin\gamma \\ -\cos\gamma \end{Bmatrix} \end{matrix}\right\} \text{SCALE AND ROTATION}$$

$$\left.\begin{aligned} E &= \xi_o \\ F &= \eta_o \end{aligned}\right\} \text{STANDARD COORDINATES OF ORIGIN}$$

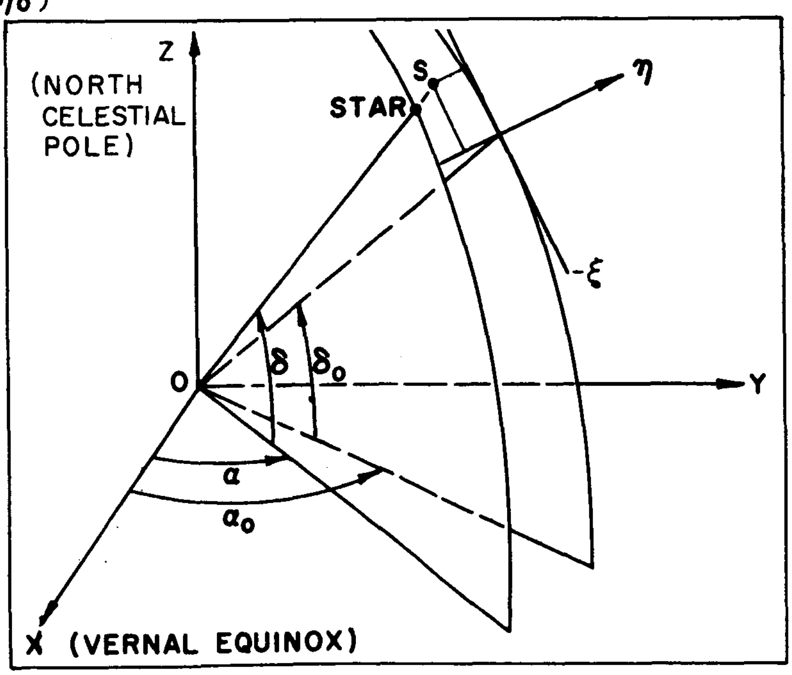
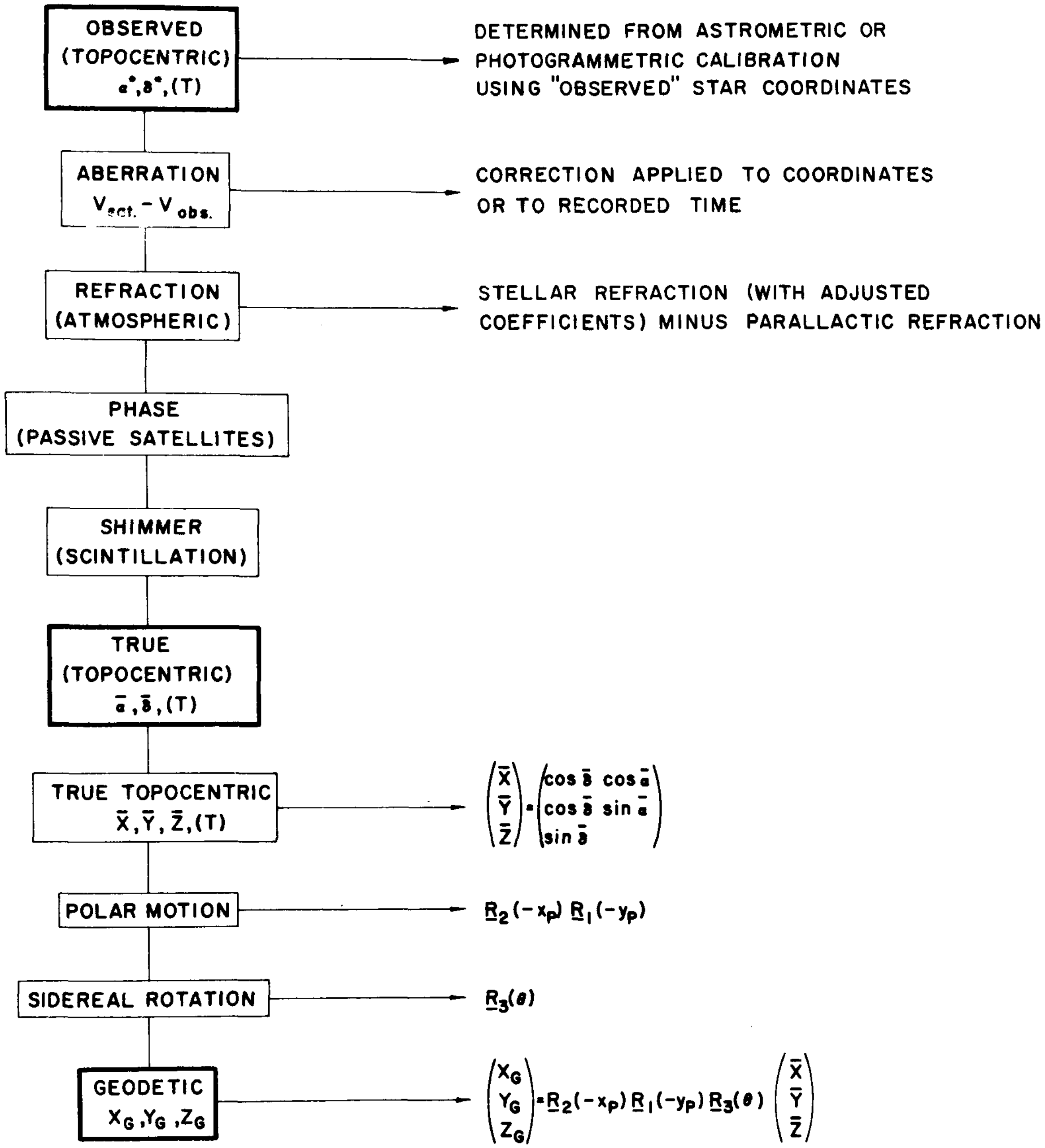



FIGURE 6

SATELLITE IMAGE CORRECTIONS




<u>NOTATION:</u>

$x_p, y_p$ COORDINATES OF THE INSTANTANEOUS POLE REFERRED TO THE I.P.M.S. 1900-05 AVERAGE TERRESTIAL POLE

$\theta$ APPARENT GREENWICH SIDEREAL TIME AT T.

FIGURE 7

PROCEDURE SUMMARY FOR MAJOR U.S. AGENCIES

| | | ACIC | | | ESSA | | | NASA | | | SAO | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| CAMERA | NAME | PC-1000 | | | BC-4 (ASTRO) | BC-4 (COSMO) | | MOTS 24 | MOTS 40 | PTH 100 | BAKER-NUNN | K-50 | |
| | FOCAL LENGTH (mm.) | 1000 | | | 305 | 450 | | 610 | 1016 | 1016 | 500 | 1000 | |
| | APERTURE (mm.) | 200 | | | 117 | | | 102 | 203 | 203 | 500 | 250 | |
| CATALOGUE | | BOSS→SAO | | | SAO | | | SAO | | | SAO | | |
| CALIBRATION | TYPE | PHOTO | | | PHOTO | | | PHOTO | | | ASTRO | | |
| | NO. OF STARS | 25-30 | | | 120 | | | 40-50 | | | 8-10 | | |
| | NO. OF SAT. IMAGES (PASSIVE) | - | | | 600 | | | - | | | 1 | | |
| | NO. OF PARAMETERS | 10 (EXT. INT: 6, REFRACT: 4) | | | 14-20 (EXT. INT: 6, DIST: 6, NON ⊥ : 1, DIFF. SC.: 1, AVAIL.: 6) | | | 8 (EXT. INT: 6, REFRACT: 2) | | | 6 | | |
| | LENS DIST. PREDETERMINED | YES | | | NO | | | YES | | | — | | |
| TIME SYNCHRONIZATION | | ACTIVE SAT. ONLY * | | | PORTABLE CLOCK & VLF | | | ACTIVE SAT. ONLY | | | PORTABLE CLOCK & VLF | | |
| | | STAR | SATELLITE | TIME | STAR | SATELLITE | TIME | STAR | SATELLITE | TIME | STAR | SATELLITE | TIME |
| STAR UPDATING AND SATELLITE IMAGE CORRECTIONS | PROPER MOTION | C | | | M | | | C | | | C | | |
| | PRECESSION | C | | | M,C | | | M | | | | | |
| | NUTATION | C | | | C | | | C | | | | | |
| | ANNUAL ABERRATION | C | | | C | | | C | | | | C | |
| | DIURNAL ABERRATION | C | | | C | C (A.S.O.) | | C | | | | | |
| M: MATRIX CORRECTION | ASTRO. REFRACTION (GARFINKEL) | CP | - CP WITH ADJ. COEF. | | CP | - CP | | CP | - CP WITH ADJ. COEF. | | IMPLICIT IN PLATE REDUCTION | | |
| C: CONVENTIONAL CORR. | PARALL. REFRACTION | | | | | C (P.S.O.) | | | C | | | | |
| CP: CONVENTIONAL DURING PLATE PROCESSING | SAT. ABERRATION (LIGHT TIME) | | | | | | C (P.S.O.) | | | | | | C TO.STA (A.S.O.) |
| P.S.O.: PASSIVE SAT. ONLY | UTC → UT1 | | | | | | C | | | | | | |
| A.S.O.: ACTIVE SAT. ONLY | UTC → A.S. | | | | | | | | | | | | C |
| | A.S. → UT1 | | | | | | | | | | | | |
| | PHASE (PASSIVE ONLY) | | | | | C | | | | | | | |

▨ : PREPROCESSING CORRECTION NEEDED

* SOME CAMERAS ARE BEING EQUIPPED WITH CHOPPING SHUTTERS

FIGURE 8

SAO PREPROCESSING FLOW CHART

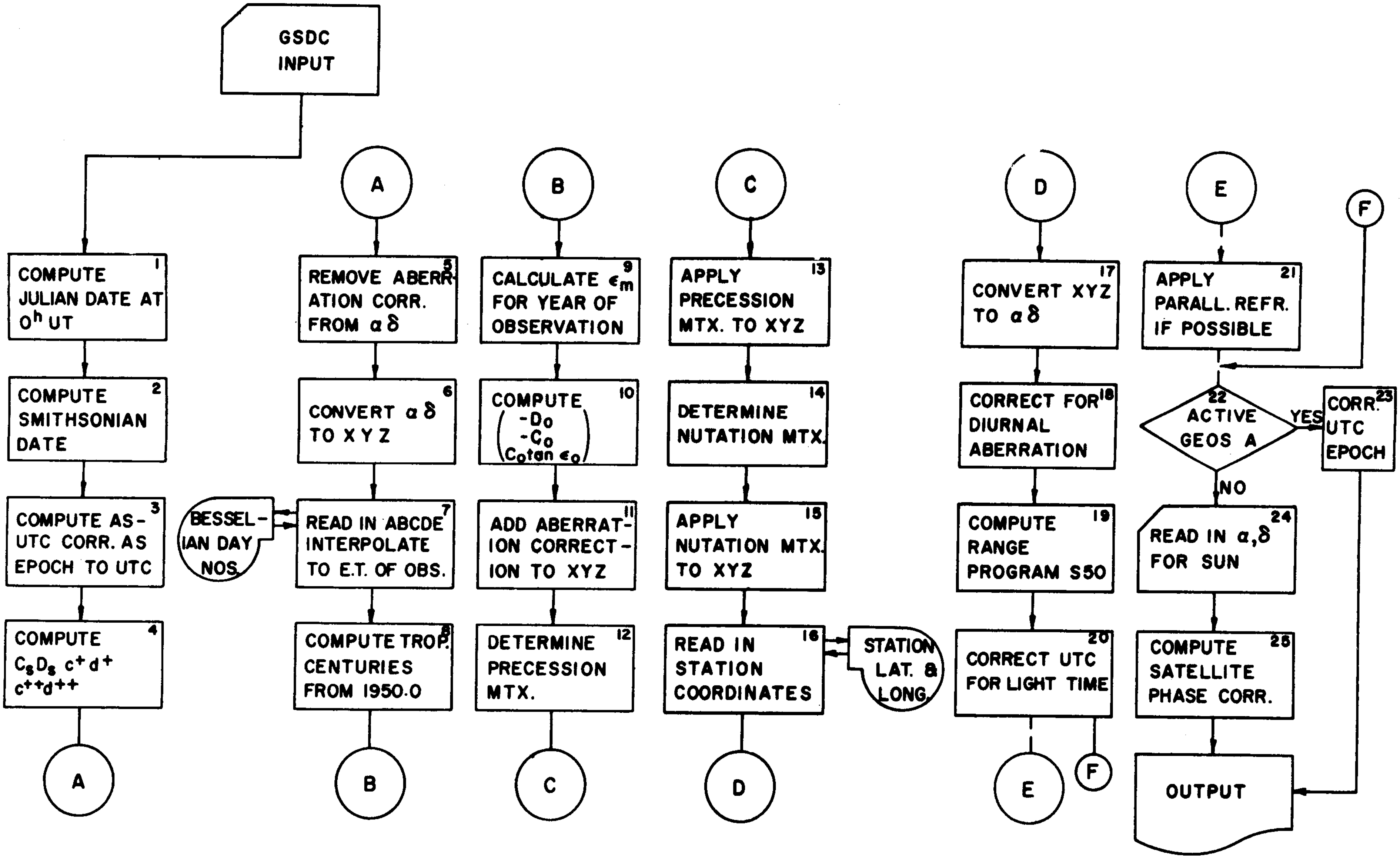



FIGURE 9

of view of this program, only SECOR data is available at the GSDS in significant quantities.

SECOR-ranges in the NASA data center (as processed by USAETL) are the results of single pass adjustments. The corrections applied are (i) zero set, (ii) tropospheric correction, (iii) ionospheric correction. The zero set correction removes ambiguities in multiples of 256 m. The tropospheric correction (TC) is computed by

$$TC = -\alpha_1 / [\sin E + (\sin^2 E + \ell^2)^{\frac{1}{2}}] \quad (1)$$

where

$\alpha_1 = 2(n_o - 1) \cdot H_o$

$n_o$ = index of refraction (ground level at observer)

$H_o$ = 7200 m (height of troposphere)

$E$ = altitude of satellite

$\ell^2 = 4\ H_o / \gamma_o$

$\gamma_o$ = geocentric distance of the observer

If two-frequency data (good, not noisy data) is available, the following ionospheric correction (IC) is computed:

$$IC = .7125\ [(D_1 - I_c) + BIC - AIC] \quad (2)$$

where

$(D_1 - I_c)$ = the difference in readings of the two-frequency data

AIC = calibration value for the VF channel (computed from pre- and post-calibration information)

BIC = calibration value for the VFIC channel (also computed from pre- and post-calibration information)

If the two-frequency data is not available, the IC is computed as follows:

$$IC = 2 / (\cos Z_1 + \sqrt{\cos^2 Z_1 + B_2 \sin^2 Z_1}) \quad (3)$$

where

$B_2 = 416667 / (R + 200,000)$

$\sin^2 Z_1 = [(1 - \sin^2 E)\ R^2] / (R + 200,000)^2$

$\cos^2 Z_1 = 1 - \sin^2 Z_1$

$R$ = range in meters

No approximations are needed for any of these corrections. The only approximations that are necessary are the satellite coordinates and velocity components ($X$, $Y$, $Z$, $\dot{X}$, $\dot{Y}$, $\dot{Z}$) at any selected epoch. These are the only parameters in the single pass least squares adjustment (which is essentially an orbit determination).

After the orbit has been determined, the orbital elements are constrained, and a range is computed from each tracking station at every one-second interval (this is a variable option). If the computed ranges agree (within a reasonable limit, which is also a variable option) with the corrected observed ranges, the data is deposited in the GSDS.

### 3.23 The Adjustment at The Ohio State University

#### 3.231 General

The system of the least squares adjustment is shown in Fig. 10. After preprocessing, the topocentric right ascensions and declinations are assumed to be free of systematic errors and are referred to the true equator and equinox of the epoch of the observation (UT1). Similarly, the topocentric ranges are also supposed to be free of systematic errors. The adjustment system is composed of three main parts:

(i) Formation of normal equations for optical or range data.

(ii) Addition of different groups of normal equations for optical or range data.

(iii) Solution of normal equations.

Four separate computer programs are involved—two for the formation of the normal equations, and one each for the addition and the solution.

#### 3.232 Formation of Normal Equations for Optical Observations

<u>Formulation</u>. The general form of the normal equations is

$$NX_g + U_g = 0$$

where $N$ is the symmetric coefficient matrix whose diagonal is composed of the $3 \times 3$ matrices,

$$N_{kk} = \sum_j M_{kj}^{-1} + P_k - \sum_j M_{kj}^{-1} \left(\sum_i M_{ij}^{-1}\right)^{-1} M_{kj}^{-1} , \tag{4}$$

while its off-diagonal portion is composed of the $3 \times 3$ matrices,

$$N_{kl} = -\sum_j M_{kj}^{-1} \left(\sum_i M_{ij}^{-1}\right)^{-1} M_{lj} . \tag{5}$$

CONCEPT FLOW DIAGRAM OF ADJUSTMENT SYSTEM

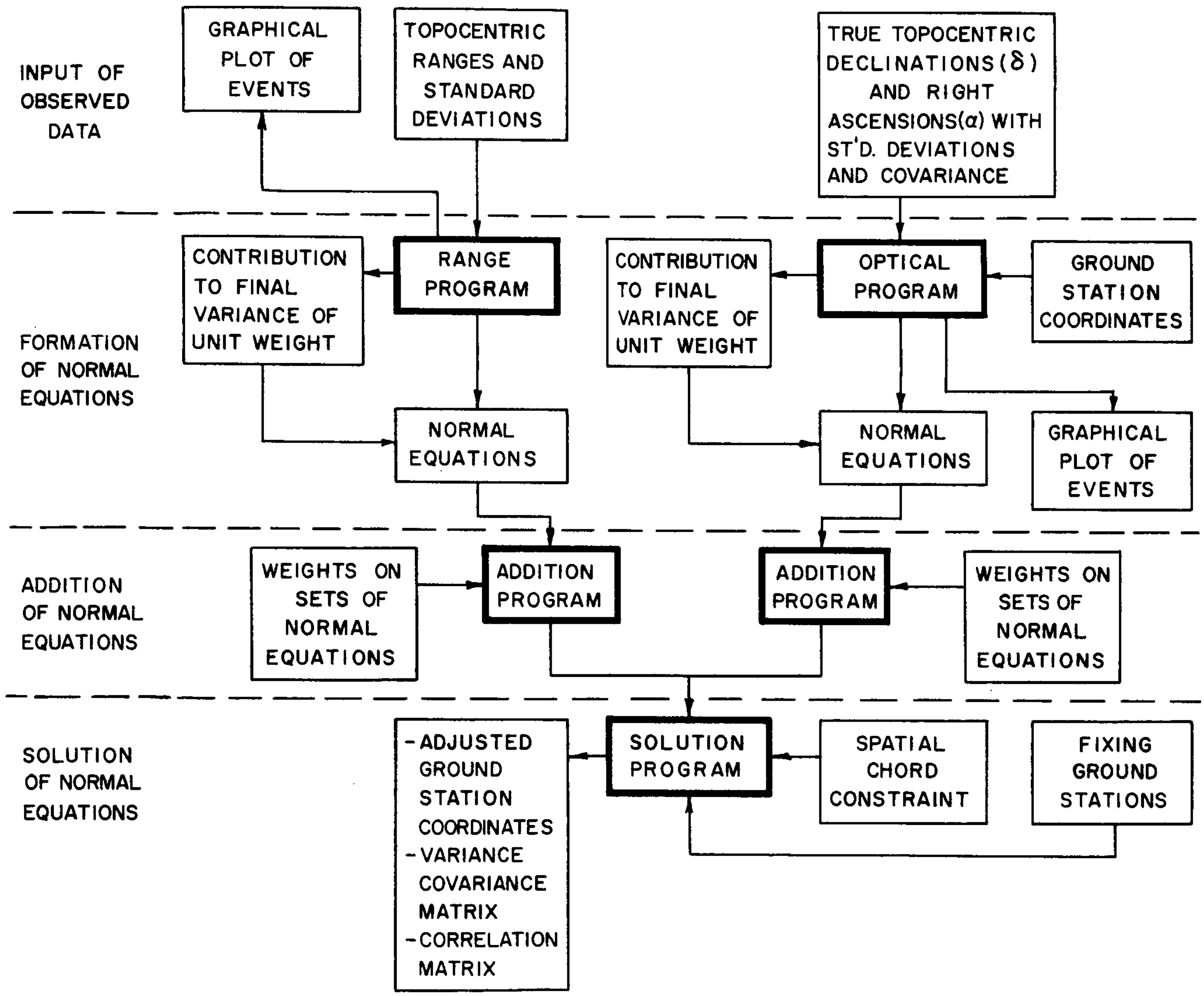



FIGURE 10

$X_g$ is the vector of unknown corrections to the preliminary Cartesian station coordinates; $U_g$ is the vector of constant terms which is composed of the $3 \times 1$ vectors,

$$U_k = -\sum_j M_{kj}^{-1} [\bar{X}_k^o - (\sum_i M_{ij}^{-1})^{-1} \sum_i M_{ij}^{-1} \bar{X}_i^o] \ . \qquad (6)$$

In the equations above, the subscripts have the following meanings: k and l denote particular ground stations; j is a particular simultaneous event; i is any ground station participating in an event j; $\sum_i$ is the summation over all ground stations involved in event j; $\sum_j$ is the summation over all events observed by ground stations k and/or l; also

$$M_{ij} = B_{ij} P_{ij}^{-1} B_{ij}' \ ,$$

where $P_{ij}$ is the $3 \times 3$ weight matrix of any observed direction, and

$$B_{ij} = R_2(-x_p)\, R_1(-y_p)\, R_3(\theta)\, R_3(-\alpha)\, R_2(-90^\circ+\delta) \begin{bmatrix} 1 & 0 & 0 \\ 0 & -\cos\delta & 0 \\ 0 & 0 & -1 \end{bmatrix} ,$$

further $P_k$ is the $3 \times 3$ weight matrix associated with a particular ground station, and $\bar{X}_i^o$ is the preliminary rectangular coordinate vector of any ground station.

The computation of equation (4) - (6) forms the core of the computer program. The addition of independent sets of normal equations is straightforward and has the advantage that batches of optical data can be adjusted separately or as part of an "accumulated" adjustment.

The Computer Program. The input to the program consists of the approximate station coordinates and the observations which are grouped according to simultaneous events. The output consists of a compacted set of normal equations punched on cards. The compacting is such that a diagonal $3 \times 3$ matrix (referred to station k) is followed immediately in the row only by those non-zero $3 \times 3$ matrices which are referred to stations co-observing with station k.

The capacity of the computer program is limited only by the total number of stations, the maximum being 150. There is no restriction on the number of simultaneous events because of the summation form of the normal equations.

In a study consisting of 40 ground stations (120 unknowns), execution time for the formation of the normal equations on the IBM 7094 was 1.9 minutes.

### 3.233 Formation of Normal Equations for Range Observations

The general form of the normal equations is the same as before,

$$NX_g + U_g = 0$$

where N is the symmetric coefficient matrix whose diagonal is composed of the $3 \times 3$ matrices,

$$N_{kk} = \sum_j a'_{kj} p_{kj} a_{kj} - \sum_j a'_{kj} p_{kj} a_{kj} [\sum_i a'_{ij} p_{ij} a_{ij}]^{-1} a'_{kj} p_{kj} a_{kj} \; ; \tag{7}$$

while its off-diagonal portion is composed of the $3 \times 3$ matrices,

$$N_{ki} = -\sum_j [a'_{kj} p_{kj} a_{kj} \; (\sum_i a'_{ij} p_{ij} a_{ij})^{-1} a'_{ij} p_{ij} a_{ij}] \, . \tag{8}$$

$X_g$ is the vector of unknown corrections; $U_g$ is the vector of constant terms which is composed of the $3 \times 1$ vectors,

$$U_k = -\sum_j a'_{kj} p_{kj} \bar{v}_{kj} \, . \tag{9}$$

The subscripts and symbols in equations (7) - (9) have the same meaning as before except in the following: $p_{ij}$ is the weight of any observed range,

$$a_{ij} = \left[ \frac{u_j^\circ - u_i^\circ}{r_{ij}^\circ}, \frac{v_j^\circ - v_i^\circ}{r_{ij}^\circ}, \frac{w_j^\circ - w_i^\circ}{r_{ij}^\circ} \right] ,$$

$u^\circ$, $v^\circ$, $w^\circ$ are the approximate Cartesian coordinates in the average terrestrial coordinate system; $\bar{v}_{kj}$ is the residual of any observed range from a particular station (resulting from a preliminary least square adjustment of any simultaneous event with the stations held fixed).

All comments about the computer program made under section 3.232 also apply to the ranging case, except that the maximum number of stations is slightly higher.

### 3.234 Solution of Normal Equations

<u>Formulation.</u> The reduction of N and U is carried out as follows (all quantities are either $3 \times 3$ matrices or $3 \times 1$ vectors):

$$\left. \begin{aligned} \bar{n}_{ij} &= \bar{n}_{ij} - \bar{n}'_{ki} \bar{n}_{kk}^{-1} \bar{n}_{kj} \\ \bar{u}_i &= \bar{u}_i - \bar{n}'_{ki} \bar{n}_{kk}^{-1} \bar{u}_k \end{aligned} \right\} \quad \begin{aligned} k &= 1, 2, \ldots, n \\ i &= k+1, k+2, \ldots, n \\ j &= i, i+1, \ldots, n \end{aligned}$$

and further

$$\begin{aligned} \bar{\bar{n}}_{ij} &= I \, , && j = i \, , \\ \bar{\bar{n}}_{ij} &= \bar{n}_{ij}^{-1} \bar{n}_{ij} \, , && j = i+1, i+2, \ldots, n \, , \\ \bar{\bar{u}}_i &= \bar{n}_{ii}^{-1} \bar{u}_i \, , && i = 1, 2, \ldots, n \, . \end{aligned}$$

The back solution for $X_g$ is

$$X_i = \sum_{k=i+1}^{n} \bar{\bar{n}}_{ik} X_k + \bar{\bar{u}}_i \; .$$

The formation of $N^{-1}$ is

$$n^{ij} = \sum_{k=i+1}^{n} \bar{\bar{n}}_{ik} n^{kj} + \delta_{ij} \bar{\bar{n}}_{ii}^{-1} \, ,$$

where $\delta_{ij} = 0$ for $i \neq j$; $\delta_{ij} = 1$ for $i = j$; and $n^{ij} = (n^{ji})'$.

The reduction, back solution, and the formation of the inverse is the core of this computer program.

The Computer Program. Two features peculiar to the program are:

(i) The coefficient matrix N is broken down into $3 \times 3$ submatrices, and similarly the U vector is treated as composed of $3 \times 1$ vectors.

(ii) The coefficient matrix N, its reduced counterpart $\bar{N}$, and $N^{-1}$ are compacted so that $3 \times 3$ zero submatrices are neither stored nor used in the computation.

The first feature is achieved rather naturally; it is because of the form of expressions (4) - (9) which are used to build up N and $U_g$. On the other hand, the second feature is achieved through programming logic. Specifically, a first matrix L is used to tag each $3 \times 3$ non-zero submatrix of N with a row and column number. A second matrix F with a one-to-one correspondence to the first is then employed to tag the storage assigned to the particular $3 \times 3$ submatrix. The individual elements of the $3 \times 3$ submatrices are all stored in one large linear array E. For example, consider

$$L = \begin{array}{c} (1) \\ (2) \\ (3) \\ (4) \\ (5) \\ (6) \\ (7) \\ (8) \end{array} \left[ \begin{array}{ccccc} 2 & 3 & & & \\ 3 & 5 & 7 & 9 & \\ 4 & 5 & 6 & 7 & 8 \\ 7 & 8 & & & \\ 5 & 7 & 8 & & \\ 7 & 8 & & & \\ 8 & & & & \\ & & & & \end{array} \right]$$

as depicting eight ground stations (listed along the left-hand side of the matrix) involved in a series of simultaneous events. The information reads as follows: Station (1) has at some time been involved with 3, 5, 7, and 9, and so on. So for L(3,5) = 8, the 9 elements beginning with cell E (F (3,5)) are the elements of $N_{38}$, the $3 \times 3$ non-zero submatrix on row 3, column 8 of the coefficient matrix N.

The reduced elements of N are stored in the locations previously created for elements in N. During reduction additional $3 \times 3$ matrices arise in locations where there were none originally in N—thus "drag storage" must be assigned. In doing so, the guide matrix L and the storage tagging matrix F are updated to account for these additional matrices. Similar drag storage is also determined during the formation of the inverse $N^{-1}$.

Once the drag storage is determined, the reduction, back solution, and inverse determinations are guided by L, the storage located by F, and the elements to be used in the computation found in E.

The capacity of the computer program is determined by two factors—the total number of stations and the amount of drag storage created during reduction and inverse formation. The latter factor may be kept at a minimum by proper ordering of the ground stations in the normal equations. Thus the maximum number of ground stations is also around 150.

In a study consisting of 40 ground stations (120 unknowns) execution time on the IBM 7094 was 1.8 minutes; this included the determination of all correlation coefficients.

## 4. PLANS FOR THE CONTINUATION OF THE PRESENT INVESTIGATION UNDER NASA GRANT NO. NGR 36-008-093

The main objectives of OSU participation in the National Geodetic Satellite Program for the time period 1967-1969 in connection with the fulfillment of the primary objectives listed in section 3.1 are the following:

(1) The extension of the computer programs for range and range rate, Doppler, and laser systems.

(2) The development of data preprocessing systems to handle the data from the above observation methods.

(3) The analysis of observational data for the National Geodetic Satellite Program for the purpose of fulfilling the primary objectives listed in section 3.1 as observational data becomes available.

(4) The development of computer programs for the adjustment of optical, ranging

and Doppler data in the short-arc mode.

(5) Theoretical investigations according to the following outline:

The objective is satellite triangulation and trilateration in combination with absolute terrestrial directions and terrestrial distances.

In satellite triangulation absolute satellite directions—topocentric right ascensions and declinations—are simultaneously observed from several ground stations. The adjustment of these observations in a given network requires that one ground station be held fixed, and the scale be obtained say from terrestrial distances and introduced via a terrestrial chord constraint between any two ground stations. It is clear that the orientation of the network is determined via the absolute satellite directions implicit in the topocentric right ascensions and declinations.

On the other hand, satellite trilateration involves the observation of topocentric ranges from several ground stations. The adjustment of these observations requires that one ground station be held fixed and the orientation be accomplished by constraining two absolute terrestrial directions within the network. The aforementioned terrestrial directions may be a result of many stations or from terrestrial vertical angle and astronomic azimuth observations. Scale is implicit in the ranges themselves; however, scale may also be introduced by terrestrial bases.

It is a natural tendency to combine absolute satellite directions, ranges, absolute terrestrial directions, and terrestrial distances together in one adjustment.

In order to determine under what circumstances these different types of observables compliment one another the best, a sequential analysis will be developed. Sequential analysis is "the continuous updating or periodic modification of an original adjustment"; more specifically, it deals with determining the effect that new observations and/or developments are planned (all containing sequential modification due to spatial chord and absolute terrestrial direction constraints), namely:

—Sequential modification of a satellite triangulation adjustment by additional absolute satellite directions.

—Sequential modification of a satellite trilateration adjustment by additional ranges.

—Sequential modification of a satellite triangulation adjustment by ranges.

—Sequential modification of a satellite trilateration adjustment by absolute satellite directions.

In addition to the items above, the following topics will also be investigated:

—Weighting ground stations whose coordinates are given on the same and/or different geodetic datums.

—Statistical tests all of which give the basis for the method of analysis at the end of each stage of the sequential modification.

—Design of test adjustments using generated fictitious data and actually observed data.

—Analysis of problems related to the combined sequential systems and hypothesis testing.

## 5. PERSONNEL

The summary which follows indicates in percentages the time worked by the project personnel during each quarter.

| Year: | 1965 | | | 1966 | | | | 1967 | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Quarter: | 2 | 3 | 4 | 1 | 2 | 3 | 4 | 1 | 2 | 3 |
| Ivan I. Mueller<br>Principal Investigator | 15% | 25% | 17% | 17% | 20% | 48% | 30% | 13% | 10% | 17% |
| Richard H. Rapp<br>Research Associate | 13 | 16 | 20 | 20 | 20 | 17 | 7 | 8 | 6 | 6 |
| Secretary | 50 | 30 | | 50 | 85 | 100 | 100 | 30 | | 27 |
| Edward J. Krakiwsky<br>Research Assistant | | 50 | 50 | 50 | 50 | 75 | 75 | 75 | 75 | 66 |
| John R. Miller<br>Technical Assistant | | 14 | 50 | 50 | 47 | 42 | 36 | 32 | 45 | 13 |
| Jeanne C. Preston<br>Technical Assistant | | 75 | 75 | 75 | 75 | 100 | 50 | 100 | 100 | 85 |
| Hans D. Preuss<br>Research Assistant | | 85 | 50 | 50 | | | | | | |
| Jack M. Ferrier<br>Technical Assistant | | | | | 6 | | 62 | 17 | | |
| Allen J. Pope<br>Research Assistant | | | | | | 30 | 50 | 50 | | |
| Arthur Corsano<br>Technical Assistant | | | | | | 6 | 14 | 5 | | |
| Georges Blaha<br>Research Assisant | | | | | | | | 30 | 30 | 13 |
| Marshall Kurfiss<br>Technical Assistant | | | | | | | 13 | | | |
| Frank M. Hotter<br>Research Assistant | | | | | | | woc | | | |

For the Department of Geodetic Science

Ivan I. Mueller

Project Supervisor

11. 29. 1967

Date

For the Ohio State University Research Foundation

Robert C. Stephenson

Executive Director

11-29-67

Date